Indian J.Sci.Res. 21 (1): 43-50, 2018 ISSN: 0976-2876 (Print)
ISSN: 2250-0138 (Online)

AVAILABILITY AND BEHAVIORAL ANALYSIS OF A MILK SYSTEM IN A DAIRY PLANT USING RPGT

SUMIT BAMAL[a1] AND RITIKESH KUMAR[b]

[ab]Department of Mathematics, Gov. Girls College, Sector 14, Gurugram, Haryana, India

ABSTRACT

In this paper behavioral analysis of a single unit system under going degradation after complete failure using Regenerative Point Graphical Technique (RPGT) is discussed. Initially the unit is working at full capacity which may have two types off ailures, one is direct and second one is through partial failure mode. There is a single server (repairman) who inspects and repairs the unit on each failure. Oncomplete failure the unit cannot be restored to its original capacity. On each repair unit under goes degradation if the server reports that unit is not repairable then it is replaced by a new one, which follows a general distribution. Fuzzy concept is used to determine failure/working state of a unit. Taking failure rates exponential, repair rates general and taking into consideration various probabilities, a transition diagram of the system is developed to determine Primary circuits, Secondary circuits & Tertiary circuits and Base state. Problem is formulated and solved using RPGT to determine system parameters. System behavior is discussed with the help of graphs and tables.

KEYWORDS: Availability, Reliability, Primary Circuits, Secondary Circuits, Tertiary Circuits, Degraded state, Regenerative Point Graphical Technique (RPGT), Mean Time to System Failure, Busyperiod of the Server, Expected No. of Server's Visits, Fuzzy Logic, Steady State.

In this paper the reliability model for availability analysis of one unit redundant system with imperfect switch over device for a milk system in a dairy plant is developed. Here main unit A is milk supply unit. When the main unit 'A' fails the nonidentical standby unit 'B' is switched in with the help of an imperfect switch. Such situation occurs in almost all the industrial units having stand-by units whether in cold stand-by, warm stand-by or hot stand-by. Initially, main unit A is operative and another non – identical unit B is kept in cold stand-by mode with imperfect switch-over device. If main unit fails, the standby unit is switched in provided the switch is working properly. The system works in reduced capacity when the main unit fails and standby unit is switched in. it has been assumed that all other processing units don't fail. If the switch is not working properly or switch is failed, then the switch will have to be repaired first or will have to be replaced by new one. The failure rates and repair rates of the main unit, stand-by and the switch are taken exponential. Using above model expression for four parameters namely Mean Time to System Failure, Availability, Busy Period of the Server and Number of Server's Visits have been determined. Using derivatives it is proved that Availability and MTSF increase with increase in repair rates and decrease with increase in failure rates while Busy Period of the Server and Number of Server's Visits increase with increase in failure rates and decrease with increase in repair rates which are in agreement with the hypothesis. Thus, this work focuses on increasing availability of the units which is helpful to the manufacturer in particular and common man in general.

ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are taken:

1. A single repair facility is available.
2. The distributions of failure times and repair times are exponential and general respectively and also different. Failures and repairs are statistically independent.
3. Repair is imperfect and repaired system is not good as new one on complete failure.
4. Nothing can fail when the system is in failed state.
5. The system is discussed for steady-state conditions.
6. Replacement of Un-repairable unit and repair facility is immediate.

[1]Corresponding author

$\overline{cycle}$: A circuit formed through un-failed states.

m-cycle : A circuit (may be formed through regenerative or non-regenerative / failed state) whose terminals are at the regenerative state m.

m-$\overline{cycle}$: A circuit (may be formed through un-failed regenerative or non-regenerative state) whose terminals are at the regenerative m state.

$\left(i \xrightarrow{S_r} j\right)$: r-th directed simple path from i-state to j-state; r takes positive integral values for different paths from i-state to j-state

$\left(\xi \xrightarrow{sff} i\right)$: A directed simple failure free path from ξ -state to i-state.

Vm,m : Probability factor of the state m reachable from the terminal state m of the m-cycle.

V$\overline{m, m}$: Probability factor of the state m reachable from the terminal state m of the m-$\overline{cycle}$.

Ri (t) : Reliability of the system at time t, given that the system entered the un-failed regenerative state 'i' at t = 0.

Ai (t) : Probability of the system in up time at time 't', given that the system entered regenerative state 'i' at t = 0.

Bi (t) : Reliability that the server is busy for doing a particulars job at time 't'; given that the system entered regenerative state 'i' at t = 0.

Vi (t) : The expected no. of server visits for doing a job in (o,t] given that the system entered regenerative state 'i' at t = 0. ',' denote derivative

Wi (t) : Probability that the server is busy doing a particular job at time t without transiting to any other regenerative state 'i' through one or more non-regenerative states, given that the system entered the regenerative state 'i' at t = 0.

μ_i : Mean sojourn time spent in state i, before visiting any other states;

ni : Expected waiting time spent while doing a given job, given that the system entered regenerative state 'i' at t=0; $\eta_i = W_i^*(0)$.

Taking into consideration the above assumptions and notations the Transition Diagram of the system is given in Figure .

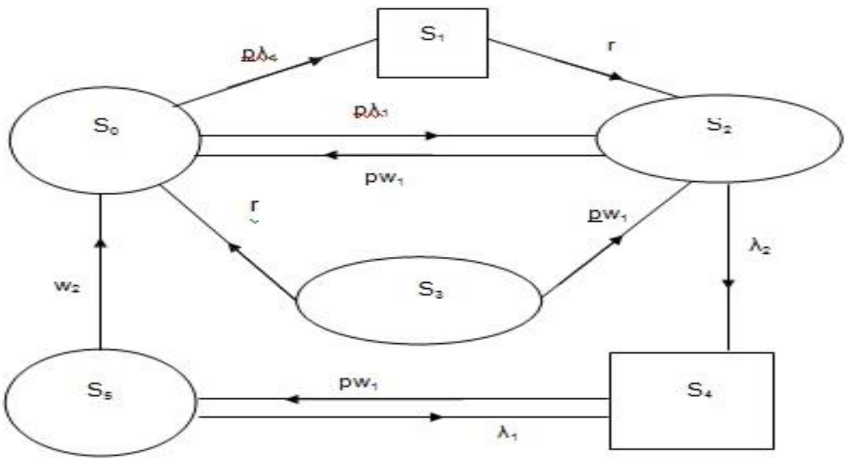

Figure 1.1

Analysis of States

State	Symbol	Model
Regenerative State/Point	⊙	0-5
Up-state	○	0,5
Failed State	▭	1,4
Reduced State	⬭	2,3

Table 1: Primary, Secondary & Tertiary Circuits at the various vertices.

Vertex i	Primary Circuits (CL1)	Secondary Circuits (CL2)
0	(0,1,2)	Nil
1	(1,2,0)	Nil
2	(2,0,1)	Nil

From the table 1 we see that at working state '0' there are maximum number of primary circuits, hence state '0' is the base state.

Table 2: Primary, Secondary, Tertiary Circuits w.r.t. the Simple Paths (Base-State '0')

Vertex j	$\left(0 \overset{S_r}{\rightarrow} j\right)$: (P0)	(P1)
0	$\left(\mathbf{0} \overset{S_1}{\rightarrow} \mathbf{0}\right)$:{0,1,0}	Nil
1	$\left(\mathbf{0} \overset{S_1}{\rightarrow} \mathbf{1}\right)$:{0,1}	Nil
2	$\left(\mathbf{3} \overset{S_1}{\rightarrow} \mathbf{2}\right)$:{0,1,2}	Nil

Table 3: Transition Probabilities

$q_{i,j}^{(t)}$	$P_{ij} = q^*_{i,j}{}^{(t)}$
$\boldsymbol{q_{0,1}} = \boldsymbol{\lambda_1 e^{-\lambda_1 t}}$	$\boldsymbol{p_{0,1}} = 1$
$\boldsymbol{q_{1,2}} = \boldsymbol{\lambda_2 e^{-\lambda_2 t}}$	$\boldsymbol{p_{1,2}} = 1$
$\boldsymbol{q_{2,0}} = \boldsymbol{w_1 e^{-w_1 t}}$	$\boldsymbol{p_{2,0}} = 1$

Table 4: Mean Sojourn Times

$R_i(t)$	$\mu_i = R_i^*(0)$
$\boldsymbol{R_0^{(t)}} = \boldsymbol{e^{-\lambda_1 t}}$	$\mu_0 = 1/\lambda_1$
$\boldsymbol{R_1^{(t)}} = \boldsymbol{e^{-\lambda_2 t}}$	$\mu_1 = 1/\lambda_2$
$\boldsymbol{R_2^{(t)}} = \boldsymbol{e^{-w_1 t}}$	$\mu_2 = 1/w_1$

Evaluation of Parameters

The Mean time to system failure and all the key parameters of the system under steady state conditions are evaluated, applying Regenerative Point Graphical Technique (RPGT) and using '0' as the base-state of the system as under:

The transition probability factors of all the reachable states from the base state 'ξ' = '0' are:

Probabilities from state '0' to different vertices are given as

$V_{0,0} = (0,1,2,0) = p_{0,1}\, p_{1,2} p_{2,0} = 1$

$V_{0,1} = (0,1) = p_{0,1} = 1$

$V_{0,2} = (0,1,2)$

$= p_{0,1}\, p_{1,2} = 1$

MTSF(T_0)

The regenerative un-failed states to which the system can transit (initial state '0'), before entering any failed state are: 'i' = 0, 1, taking 'ξ' = '0'.

$$\text{MTSF } (T_0) = \left[\sum\nolimits_{i,sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr(sff)} i\right)\right\}\mu i}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \div \left[1\text{-}\sum\nolimits_{sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr(sff)} \xi\right)\right\}}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$$= (V_{0,0}\mu_0+V_{0,1}\mu_1)/(V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2)$$

Availability of the System

The regenerative states at which the system is available are 'j' = 0, 1 and the regenerative states are 'i' = 0, 1, 2 taking 'ξ' = '0' the total fraction of time for which the system is available is given by

$$A_0= \left[\sum\nolimits_{j,sr}\left\{\frac{\{pr(\xi^{sr\to} j)\} fj,\mu j}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \div \left[\sum\nolimits_{i,s_r}\left\{\frac{\{pr(\xi^{sr\to} i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$$A_0 = \left[\sum\nolimits_j V_{\xi,j}\, , f_j, \mu_j\right] \div \left[\sum\nolimits_i V_{\xi,i}\, , f_j, \mu_i^1\right] = (V_{0,0}f_0\mu_0+V_{0,1}f_1\mu_1)/(V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2)$$

Proportional Busy Busy Period of the Server

The regenerative states where server 'j' = 2 and regenerative states are 'i' = 0 to 2taking ξ = '0', the total fraction of time for which the server remains busy is

$$B_0= \left[\sum\nolimits_{j,sr}\left\{\frac{\{pr(\xi^{sr\to} j)\}, nj}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \div \left[\sum\nolimits_{i,s_r}\left\{\frac{\{pr(\xi^{sr\to} i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$$= \left[\sum\nolimits_j V_{\xi,j}\, , n_j\right] \div \left[\sum\nolimits_i V_{\xi,i}\, , \mu_i^1\right]$$

$$= (V_{2,0}\mu_2)/(V_{0,0}\, \mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2)$$

Expected Number of Inspections by the Repair Man

The regenerative states where the repairman does this job j = 2 the regenerative states are i = 0 to 2, Taking 'ξ' = '0', the number of visit by the repair man is given by

$$V_0= \left[\sum\nolimits_{j,sr}\left\{\frac{\{pr(\xi^{sr\to} j)\}}{\Pi_{k_1\neq\xi}\left\{1\text{-}V_{\overline{k_1 k_1}}\right\}}\right\}\right] \div \left[\sum\nolimits_{i,s_r}\left\{\frac{\{pr(\xi^{sr\to} i)\}\mu_i^1}{\Pi_{k_2\neq\xi}\left\{1\text{-}V_{\overline{k_2 k_2}}\right\}}\right\}\right]$$

$$V_0 = \left[\sum\nolimits_j V_{\xi,j}\right] \div \left[\sum\nolimits_i V_{\xi,i}\, , \mu_i^1\right]$$

$$V_0= (V_{2,0}\mu_2)/(V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2)$$

Illusteration: When failure and repair rates are equal

MTSF (T_0) =$[(1/\lambda_1)+(1/\lambda_2)]/[(1/\lambda_1)+(1/\lambda_2)+(1/w_1)]$

$= [2w/(2w+\lambda)]$

Table 5: MTSF Table

T_0	w = 0.80	w = 0.90	w = 1.00
λ = 0.50	0.761904	0.782609	0.800000
λ = 0.60	0.727272	0.750000	0.769230
λ = 0.70	0.695652	0.720000	0.740740

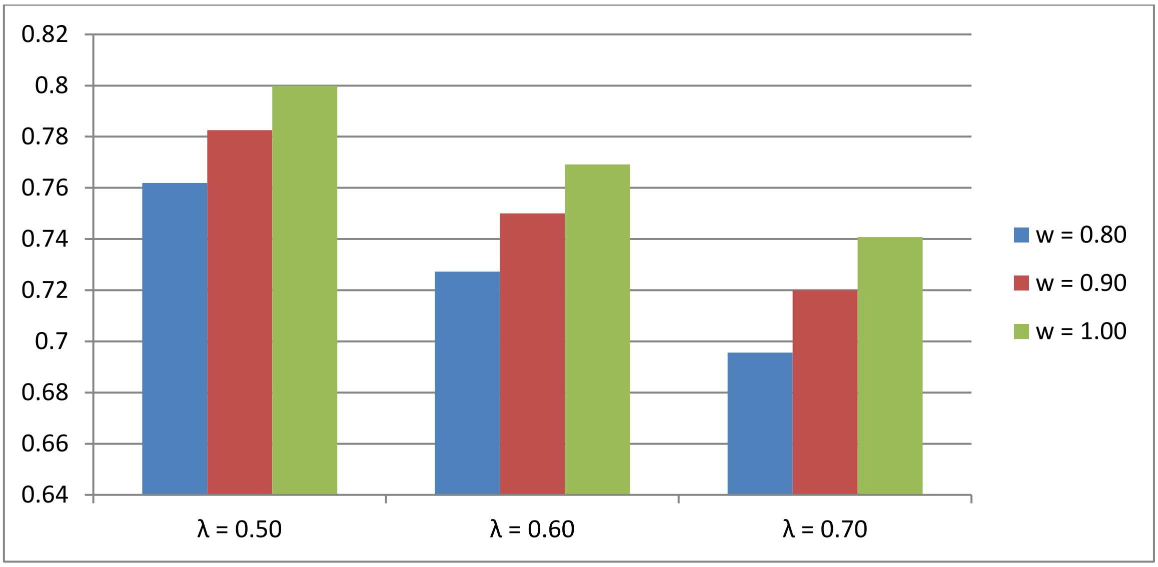

Figure 2: MTSF Graph Availability of the System (A_0)

Table 6: Availability of the System Table

A_0	w = 0.80	w = 0.90	w = 1.00
λ = 0.50	0.761904	0.782609	0.800000
λ = 0.60	0.727272	0.750000	0.769230
λ = 0.70	0.695652	0.720000	0.740740

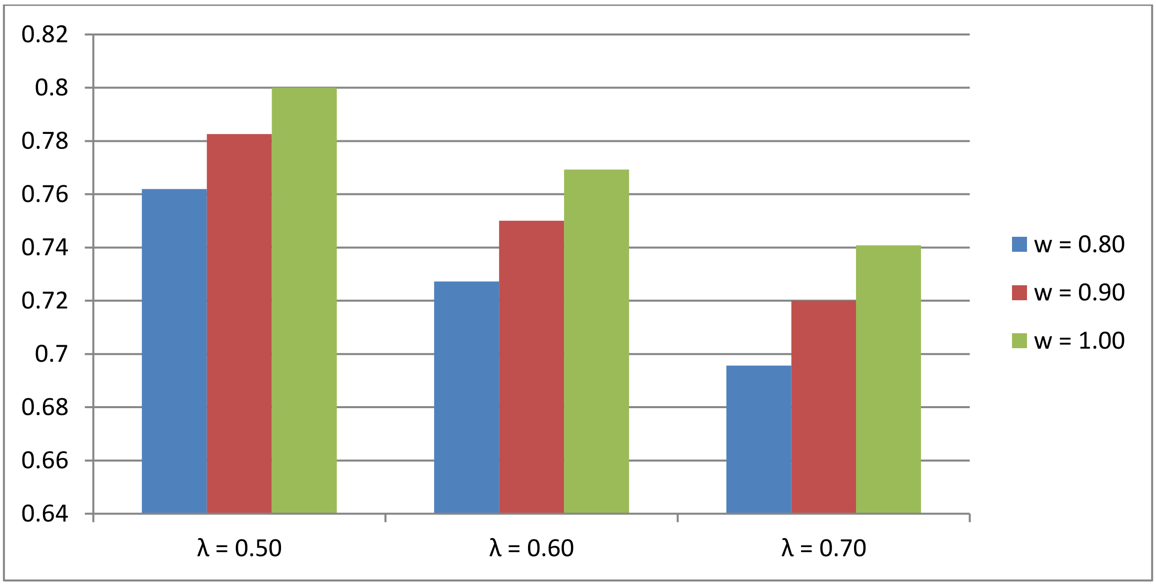

Figure 3: Availability of the System Graph

Fractional Busy Period of the Server (B_0) in Unit Time:

(B_0)= $[(1/w_1)/\{(1/\lambda_1)+(1/\lambda_2)+(1/w_1)\}]$

= $[\lambda/(2w+\lambda)]$

Table 7: Busy Period of the Server Table

B_0	w = 0.80	w = 0.90	w = 1.00
λ = 0.50	0.238095	0.217391	0.200000
λ = 0.60	0.272727	0.250000	0.230769
λ = 0.70	0.304348	0.280000	0.259259

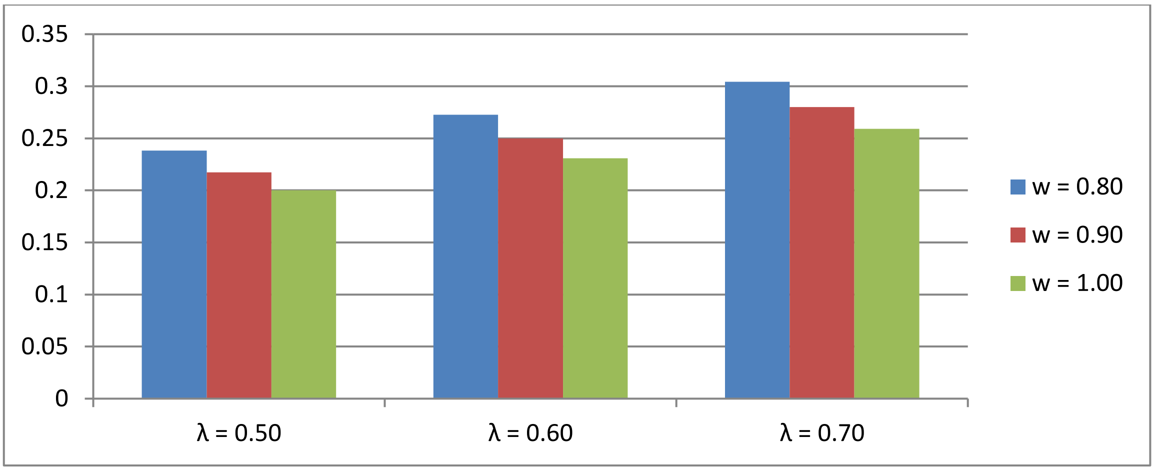

Figure 4: Busy Period of the Server Graph

Expected Fractional Number of Server's Visits (V_0) in Unit Time:

Table 8: Expected Number of Server's Visits Table

V_0	w = 0.80	w = 0.90	w = 1.00
λ = 0.50	0.238095	0.217391	0.200000
λ = 0.60	0.272727	0.250000	0.230769
λ = 0.70	0.304348	0.280000	0.259259

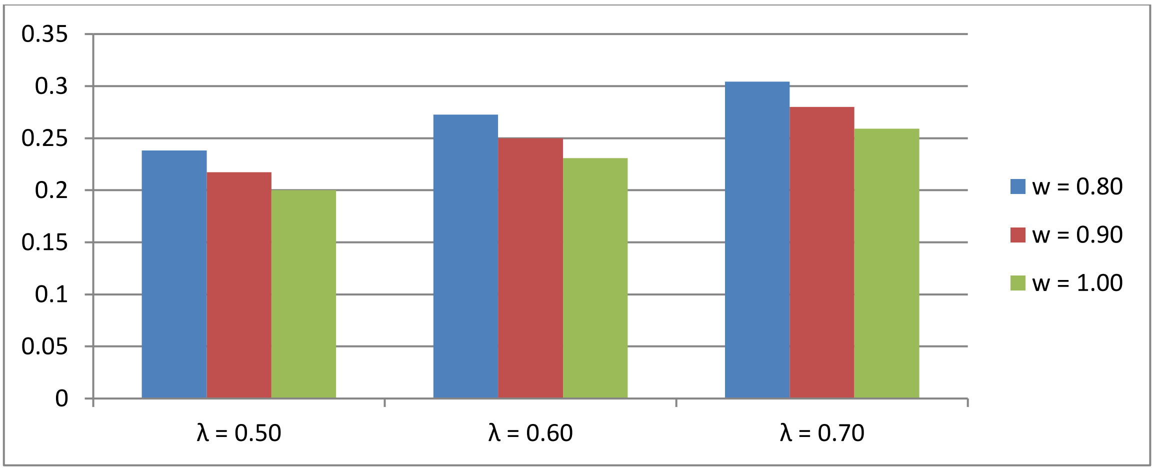

Figure 5: Expected Number of Server's Visits Graph

Profit Function

$= A_0R_0-(B_0R_1+V_0R_2)$

$= A_0R_0-B_0R_1-V_0R_2$

Where

A_0 = Availability of System

B_0 = Busy Period of Server

V_0 = Expected Number of Inspection by the Repair Man

R_0 = Revenue

R_1 = Busy Period per Unit

R_2 = Per Visit Cost

$R_0 = 1000$

$R_1 = 50$

$R_2 = 100$

Profit= $[2000w/(2w+\lambda)]-[50\lambda/(2w+\lambda)]-[100\lambda/(2w+\lambda)]$

$= [2000w-150\lambda]/[(2w+\lambda)]$

Table 9: Profit

	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	726.1905	750.0000	770.0000
λ = 0.6	686.3636	712.5000	734.6154
λ = 0.7	598.0000	678.0000	701.8519

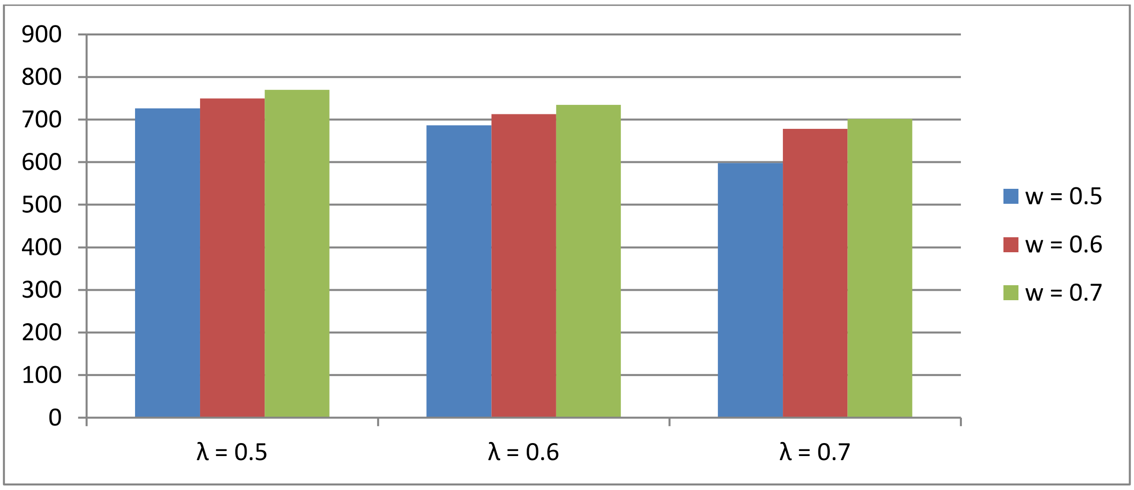

Figure 6: Profit Function Graph

CONCLUSION

From above tables and graph we see that the result obtained using Regenerative Point Graphical Technique is same as obtained by using Regenerative Point Technique and other techniques. Here, we derived the results very easily and quickly without writing any state equations and without any lengthy procedures, long calculations and simplifications. If we fix the optimum values of system parameter and know prehand the failure rates as these are beyond the control of the management, then we accertain the repair rates to achieve the optimum values for various system parameters and profit function. Results derived in corollary match with the results obtained by other researchers and practically possible and other results may also be obtained when there is no repair or no failures.

REFERENCES

Garg, R.C. (1980), 'Dependability of a Complex System Having Two Types of Components'; IEEE Trans. On Reliability, Vol. R -12.

Hutn, L.T. (1966), 'Reliability prediction technique for complex systems'; IEEE Trans. On Reliability, Vol. 15, No. 1

Graver, D.P. (1963), 'Time to failre and availability of parelledled system with repair'; IEEE Trans. On Reliability, Vol. R -12.

Branson, M.H. & Shah, B. (19781),'Reliability analysis of systems comprised of units with arbitrary repair time distributions'; IEEE Trans. On Reliability, Vol.R- 20, No. 4.

Osaki, S.(1970),'System reliability analysis by Markov renewal processes'; J.Oper., Res. Soc. Japan, Vol. 112.

Osaki,S. (1972),'On a two unit standby system with imperfect switchover';IEEE Trans. On Reliability, Vol. R- 21, No. 1.

Osaki, S. &Nakagawa, T. (1971), 'On a two unit standby redundant system with a standby failre'; Operations Research, Vol. 19, No. 2.

Osaki, S. & Nagakawa, T. (1974),'Stochastic behavior of a two unit standby redundant system'; INFOR, Vol.12.

Kumar, A.(1976),'Steady state profit in a two unit standby system'; IEEE Trans. On Reliability, Vol. R-25, No. 2.

Howard, R.A. (1971),'Dynamic probalistic sytems'; Vols. I and II, John Wiley and Sons, New York.

Liebowitz, B.H. (1966),'Reliability considerations for a two element redundant system with generalized repair times ';Ops. Res., Vol. 14, No. 2.

Nakagawa, T. (1976), 'On a replacement problem of the cumulative damager model'; operational research quarterly, Vol. 27, No. 1.

Arora, J. R. (1976), 'Reliability of a two unit priority standby redundant system with finite repair capability'; IEEE Trans. on reliability, Vol. R-25, No. 3.

Arora, J.R. (1977), 'Reliability of several standby priority redundant systems'; IEEE Trans. on reliability, Vol. R-26, No. 4.